Solutions to Workbook-2 [Mathematics] | Permutation & Combination

Level - 2	DAILY TUTORIAL SHEET 8

156.(C) Choose two women is ${}^9C_2$ ways and choose two men in ${}^9C_2$ ways. Therefore, total number of ways $= {}^9C_2 \cdot {}^9C_2 \cdot 2$

157.(A) Say x_1 of them show "1", x_2 of them show "2" and so on x_6 of them show "6"

$\Rightarrow \quad x_1 + x_2 + x_3 + \ldots + x_6 = n$

Total number of outcomes is equal to the total number of non-negative solutions

$\Rightarrow \quad {}^{n+6-1}C_{6-1} = {}^{n+5}C_5$.

158.(CD) Three numbers a, b, c are in A.P, if $2b = a + c$

L.H.S. is even so should the R.H.S.

Say n is odd, then total number of odd numbers $= \dfrac{n+1}{2}$ and total number of even numbers $= \dfrac{n-1}{2}$.

Once you pick a and c, b is fixed. Therefore, total number of A.P.s $= \left({}^{\frac{n+1}{2}}C_2 + {}^{\frac{n-1}{2}}C_2 \right) \cdot 2!$.

Find out the total number of A.P.s if n is even.

159.(BD) Let x_5 be the dummy non-negative integer which when added to the L.H.S. of the inequality $x_1 + x_2 + x_3 + x_4 \le n$ converts it into an equality. Now, $x_1 + x_2 + x_3 + x_4 + x_5 = n$.

There's a bijection between the number of solutions of the inequality and the number of solutions of the equation.

Therefore, the total number of solutions $= {}^{n+5-1}C_{5-1} = {}^{n+4}C_4 = {}^{n+4}C_n$.

160.(BC) Not all correct answers can be made in $2^{10} - 1$ ways.

(A) This can be done in $2^{10} - 2$ ways **(B)** This can be attempted in $2^{10} - 1$ ways

(C) This can be done in $2^{10} - 1$ ways **(D)** This can be done in 10 ways.

161.(BC) $${}^{2n}P_n = \frac{\lfloor 2n}{\lfloor n} = \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot -- 2n}{\lfloor n} = \frac{(2 \cdot 4 \cdot 6 \cdot 8..2n)(1 \cdot 3 \cdot 5 --)}{\lfloor n} = \frac{2^n (\lfloor n).(1 \cdot 3 \cdot 5 --.)}{\lfloor n} = 2^n.(1 \cdot 3 \cdot 5 --)$$

$= 2 \cdot 6 \cdot 10 \cdot 14 --- n$ terms

Also $${}^{2n}p_n = \frac{\lfloor 2n}{\lfloor n} = \frac{1 \cdot 2 \cdot 3 \cdot --- n \cdot (n+1)(n+2)--(n+n)}{\lfloor n} = (n+1)(n+2)(n+3)--(n+n)$$

162.(BD) **(A)** No. of ways $= \dfrac{\lfloor n-1}{\lfloor p}$ **(B)** No. of ways $= {}^{n-1}C_p$

(C) No. of ways $= {}^{n-1}C_{p-1}$ **(D)** No. of ways $= {}^{n-1}C_p$

163.(ABCD) **(A)** $3 \times 7!$

(B) Total no. of arrangements of 'r' white and $15 - r$ black balls $= \dfrac{15!}{r!(15-r)!}$

(C) Total no. of combinations $= 5 \times 6 \times 2^3 = 240$ **(D)** Total no. of selections = 35

164.(BC) $P = (n-r)....(n-2)(n-1)n(n+1)(n+2)....(n+r)$ is multiplication of $2r+1$ consecutive integers which is divisible by $(2r+1)!$.

165.(AB) $f(n) = 1! + 2! + 3! + ... + n!$

$f(n+1) = 1! + 2! + 3! + ... + (n+1)!$

$f(n+2) = 1! + 2! + 3! + ... + (n+2)!$

$f(n+2) - f(n+1) = (n+2)! = (n+2)(n+1)! = (n+2)\left[f(n+1) - f(n)\right]$

$\Rightarrow \quad f(n+2) = (n+3)f(n+1) - (n+2)f(n) \qquad \Rightarrow \qquad P(x) = x+3,\ Q(x) = -x-2$

166.(AD) Number of selections of 7 digits out of the digit 1, 2, 3, ..., 9 = $^9C_7$

Number of digits out of these 7 selected digits excluding the greatest digit = 6

These 6 digits can be divided in two groups each having 3 digits = $\dfrac{6!}{3!3!2!} = {}^6C_3 \times \dfrac{1}{2!}$

But the 3 digits on one side can go on the other side

$\therefore$ Required number of ways = $^9C_7 . {}^6C_3 . \dfrac{1}{2!} . 2! = {}^9C_7 . {}^6C_3 = {}^9C_2 . {}^6C_3$

167.(ABCD) Let the friends be $F_1, F_2, F_3, F_4, \ldots, F_8, F_9, F_{10}, F_{11}$

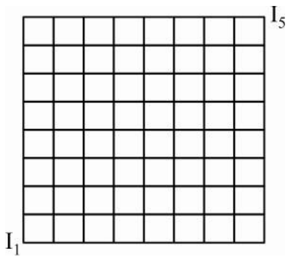
Thus, required ways = Total number of ways - Number of ways when two particular friends F_1 and F_2 (say) are invited $= {}^{11}C_{35} - n(F_1F_2 \ldots) = {}^{11}C_5 - {}^9C_3$ ways. $\Rightarrow$ (A)

Also, $n(F_1\bar{F}_2 \ldots) = {}^9C_4 + n(F_2\bar{F}_1 \ldots) = {}^9C_4 + n(\bar{F}_1\bar{F}_2 \ldots) = {}^9C_5$

$\Rightarrow \quad {}^9C_5 + 2\left({}^9C_4\right) = {}^9C_4 + 2\left({}^9C_4\right) = 3\left({}^9C_4\right)$ ways = $3 \times (126) = 378$ ways.

168.(ABCD)

Let x be the number of horizontal steps taken by I_1 and x' that by I_2 and let y be the number of vertical steps taken by I_1 and y' by I_2.



$\therefore \quad x + y = x' + y' \qquad (\because$ speed of I_1 and I_2 are equal)

Further $x + x' = 8$ and $y + y' = 8$

$\Rightarrow \quad x' = 8 - x\ ;\ y' = 8 - y$

$\therefore \quad x + (y) = (8-x) + (8-y) \Rightarrow \qquad 2(x+y) = 16 \qquad \Rightarrow \qquad x + y = 8$

$\Rightarrow \quad y = 8 - x$

$\therefore$ To meet at any point

	Horizontal	**Vertical**
I_1	x	$8-x$
I_2	$8-x$	x

Number of ways in which I_1 can take x horizontal steps and $(8-x)$ vertical steps

$= \dfrac{\left[x + (8-x)\right]!}{x!(8-x)!} = \dfrac{8!}{x!(8-x)!} = {}^8C_x$

Similarly, I_2 can take $(8-x)$ horizontal steps and x vertical in ^8C_x ways.

$\therefore$ The two can meet corresponding to a particular value of x in $^8C_x \times {}^8C_x = \left({}^8C_x\right)^2$ ways.

But $x \in \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

$\therefore$ Total number of ways in which two insects can meet during their journey.

$$= \sum_{x=0}^{8} \left({}^8C_x\right)^2 = \left({}^8C_0\right)^2 + \left({}^8C_2\right)^2 + \ldots + \left({}^8C_8\right)^2 = {}^8C_0 . {}^8C_8 + {}^8C_1 . {}^8C_7 + {}^8C_2 . {}^8C_6 + \ldots + {}^8C_8 . {}^8C_0 = {}^{16}C_8$$

$\Rightarrow$ (D) is correct option.

Further ${}^{16}C_8 = \dfrac{16\times15\times14\times13\times12\times11\times10\times9}{1\times2\times3\times4\times5\times6\times7\times8}$ $\Rightarrow$ (A) is correct.

$$= \frac{(2)^4 \times 2 \times (2)^2 (15\times7\times13\times3\times11\times5\times9)}{1\times2\times3\times4\times5\times6\times7\times8} = (2)^8 \left(\frac{1}{1}\right)\left(\frac{3}{2}\right)\left(\frac{5}{3}\right)\left(\frac{7}{4}\right)\left(\frac{9}{5}\right)\left(\frac{11}{6}\right)\times\left(\frac{13}{7}\right)\times\left(\frac{15}{5}\right)$$

$\Rightarrow$ (B) is correct.

$= \left(\dfrac{2}{1}\right)\left(\dfrac{6}{2}\right)\left(\dfrac{10}{3}\right)\left(\dfrac{14}{4}\right)\left(\dfrac{18}{5}\right)\left(\dfrac{22}{6}\right)\times\left(\dfrac{26}{7}\right)\times\left(\dfrac{30}{8}\right)$ $\Rightarrow$ (C) is correct.

Note that the number of ways in which I_1 and I_2 meet being numerically equal to number of ways in which I_1 completes its journey is not a coincidence. Every way in which I_1 and I_2 meet gives us a unique path for I_1 to complete its journey because I_1 can simply retrace the steps taken by I_2 before meeting I_1 to reach its destination.

169.(AD) Case 1: When two points from non-collinear points and two points from collinear points are taken to form a quadrilateral. Number of possible quadrilaterals $= {}^7C_2 \times {}^5C_2$

Case 2: When three points from non-collinear points and one point from collinear points are taken to form a quadrilateral.

Number of possible quadrilaterals $= {}^7C_3 \times {}^5C_1$.

Case 3: All the four points are taken from non-collinear points to form a quadrilateral.

Number of possible quadrilaterals = ${}^7C_4$

$\therefore$ Total possible quadrilaterals $= {}^7C_2 \times {}^5C_2 + {}^7C_3 \times {}^5C_1 + {}^7C_4$

$= 21\times10 + 35\times5 + 35 = 420 = 7\times6\times5\times2 = 2\left(\dfrac{7!}{4!}\right) = 2.{}^7P_3$

170.(BCD) Let the occupied seat is represented by G and vacant by R, thus we are booking for permutation of 4G and 6R such that no two G are together.

$\underbrace{(1)R\,(2)\,R(3)\,R(4)R(5)\,R(6)R(7)}$

Consider it is equal to number of ways of choosing 4 places out 7 vacant place $= {}^7C_4$

$\Rightarrow$ Number of ways of arranging 4 people in 7 is required ways $= {}^7C_4 \times 4! = {}^7C_3 \times 4! = 840$